

ST. JOSEPH'S ACADEMY, DEHRADUN
FIRST TERM EXAMINATION SEPTEMBER, 2024-25

SUBJECT: MATHEMATICS

CLASS: XII

M.M. :80 TIME : 3 HOURS

NAME OF THE STUDENT.....CLASS/SECTION.....ROLL NO.....

*(Candidates are allowed additional 15 minutes for only reading the paper.
They must NOT start writing during this time).*

Candidates are required to attempt all questions. Internal choice has been provided in one question of two mark, one question of four mark and one question of six mark.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [].

Mathematical tables and graph papers are provided.

Question 1. In subparts (i) to (xii) choose the correct options and in subparts (xiii) to (xx), answer the questions as instructed:

(i) Assertion (A): The system of equations $2x+5y=7$ and $6x+15y=13$ is inconsistent. [1]

Reason (R): The system of equation $AX = B$ is inconsistent if $|A|=0$ and $(\text{adj } A)B \neq 0$

(a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation for Assertion.

(c) Assertion is true and Reason is false.

(d) Assertion is false and Reason is true.

(ii) Find the values of x for which, $\begin{vmatrix} 3 & 2x \\ x & 12 \end{vmatrix} = \begin{vmatrix} 4 & -2 \\ 2 & 0 \end{vmatrix}$ [1]

(a) 4

(b) -4

(c) ± 4

(d) -1

(iii) The interval on which the function $f(x) = x^4 - 2x^2$ is decreasing is: [1]

(a) $(-\infty, -1) \cup (0, 1)$

(b) $(-2, -1)$

(c) $(-\infty, 1) \cup (2, \infty)$

(d) $[-1, 1]$

(iv) The rate of change of area of a circle with respects to its radius R at $R = 6\text{cm}$ is: [1]

(a) 8π

(b) 10π

(c) 11π

(d) 12π

(v) The function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(x) = 2x^3 + 5$ is: [1]

(a) one -one but not onto

(b) onto but not one-one

(c) neither one-one nor onto

(d) one-one and onto

(vi) The expression $\sin^{-1}(\frac{3}{5}) + \sin^{-1}(\frac{4}{5})$ is equal to: [1]

(a) $\cot^{-1} 1$

(b) $\pi/2$

(c) $\cot^{-1} \frac{2}{11}$

(d) $\sin^{-1} \frac{5}{11}$

(vii) Let R be the relation "is perpendicular to" on the set of all lines in a plane, then R is a: [1]

- (a) only symmetric relation
- (b) symmetric and transitive relation
- (c) symmetric relation and reflexive relation
- (d) equivalence relation

(viii) Value of $\cos^{-1}(\cos \frac{13\pi}{6})$ [1]

- (a) $\frac{13\pi}{6}$
- (b) $\pi/6$
- (c) $\frac{7\pi}{6}$
- (d) $-\pi/2$

(ix) The equation of the tangent at (2,3) on the curve $y^2 = ax^3 + b$ is $y = 4x - 15$; then the values of a and b ? [1]

- (a) 1, 6
- (b) -1, 7
- (c) 2, -7
- (d) None of these

(x) If A is a square matrix such that $2A^2 = I$, then A^{-1} is: [1]

- (a) $2A$
- (b) $4A$
- (c) A
- (d) $A + I$

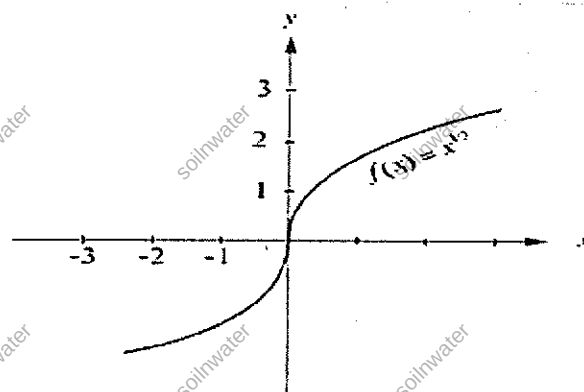
(xi) If A is a square matrix and $A + A^t = 0$, then A is: [1]

- (a) symmetric matrix
- (b) skew-symmetric matrix
- (c) singular matrix
- (d) non-singular matrix

(xii) Let R be the relation "is less than" on the set of natural numbers, then R is a: [1]

- (a) reflexive relation
- (b) symmetric relation
- (c) transitive relation
- (d) equivalence relation

(xiii) Consider the graph of $y = x^{\frac{1}{3}}$ [1]



Statement 1: The above graph is continuous at $x = 0$.

Statement 2: The above graph is differentiable at $x = 0$.

Which one of the following options is correct?

- (a) Both the statements are true.
- (b) Both the statements are false.
- (c) Statement 1 is true and Statement 2 is false.
- (d) Statement 1 is false and Statement 2 is true

(xiv) Evaluate: $\sec^{-1}(\sqrt{2}) + \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ [1]

(xv) If A and B are two matrices such that $AB=A$ and $BA=B$, then B^2 is equal to: [1]

- (a) A
- (b) B
- (c) 0
- (d) none of these

(xvi) If $y = \cot x$ then find $\frac{d^2y}{dx^2}$. [1]

(xvii) Find a point on curve $y = x^3 - 9x$ where its tangent is parallel to x-axis. [1]

(xviii) Evaluate the following integral; $\int \frac{x^2}{x^2+1} dx$ [1]

(xix) For what value of x, matrix $\begin{bmatrix} 2x & 4 \\ 3-x & 1 \end{bmatrix}$ is a symmetric matrix? [1]

(xx) Find the derivative of $2\tan^{-1} x$ w.r.t $4 \cot^{-1} x$. [1]

Question 2 $\int \frac{\cos x}{\cos x + \sin x} dx$ [2]

Question 3 If $y = \sin^2 \theta$, $x = \cos^2 \theta$, find $\frac{d^2y}{dx^2}$ OR [2]

The height of a closed cylinder is constant at 14cm but its radius is decreasing at the rate of 0.05cm/sec. How fast is its volume changing when the radius becomes 7cm?

Question 4 Find the adjoint the following matrix, $\begin{bmatrix} 7 & -2 & -8 \\ 1 & 1 & 4 \\ 1 & 5 & 6 \end{bmatrix}$ [2]

Question 5 Evaluate the following integral; $\int \frac{1}{\sin^2 x \cdot \cos^2 x} dx$ [2]

Question 6 Check the continuity of the following given function at the point $x = -2$ [2]

$$f(x) = \begin{cases} |x| + 2; & \text{if } x \leq -2 \\ -2x; & \text{if } -2 < x < 3 \end{cases}$$

Question 7 By using the properties of determinants prove the following:- [4]

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2).$$

Question 8 Let $A = \mathbb{R} - \{3/5\}$ and $B = \mathbb{R} - \{3/5\}$. Consider the function $f: A \rightarrow B$ defined by

$f(x) = \frac{3x+2}{5x-3}$. Show the f is one-one and onto and hence find f^{-1} . [4]

Question 9 Solve for x : $2 \tan^{-1}\left(\frac{1}{8}\right) + 2 \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{4}$. [4]

Question 10 If $y = (x-1) \log(x-1) - \log(x+1)^{x+1}$, then prove that $\frac{dy}{dx} = \log\left(\frac{x-1}{x+1}\right)$ [4]

OR

Differentiate $x^{\sin x} + (\sin x)^{\log x}$ w.r.t. x .

Question 11 Find the point on the curve $y^2 = 4x$ which is nearest to the point $(2, 1)$. [4]

Question 12 (a) If $y = \sin^{-1} \sqrt{\frac{1-x^2}{2}}$, using proper substitution find $\frac{dy}{dx}$ [2+4=6]

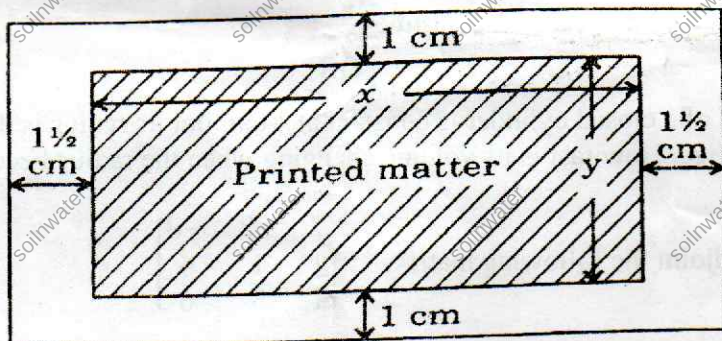
(b) find the intervals on which the function $f(x) = \sin x + \cos x$, $x \in [0, 2\pi]$ is strictly increasing or strictly decreasing.

Question 13 To raise money for a charity, students of three classes XIIA, XIIB and XIIC organised an exhibition in their school, where they sold old books, cloth bags and handicraft items. Student of XII A sold 20 old books, 10 cloth bags and 15 handicraft items and raised Rs. 660. Student of XII B sold 22 old books, 8 cloth bags and 14 handicraft items and raised Rs. 636. Student of XII C sold 18 old books, 12 cloth bags and 15 handicraft items and raised Rs. 666

Answer the following question:

- Translate the problem into a system of equations.
- Solve the system of equation by using matrix method.
- Hence, find the cost of one old book, 1 cloth bags and 1 handicraft item.

Question 14 A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1cm and the margins on the left and right are to be $1\frac{1}{2}$ cm as shown below: [6]



On the basis of the above information, answer the following questions:

- Write the area of visiting card in terms of x .
- Obtain the dimensions of the card of minimum area.

OR

A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 28 m. Find the dimensions of the window to admit maximum light through the whole opening.

Question 15 Integrate: (a) $\int \sin^4 x \, dx$.

(b) $\int \frac{x^3}{x^2-4} \, dx$

[3+3=6]

Question 16 If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then find the value of $x(x+1)^2 y_2 + (x+1)^2 y_1$ [6]